SIMONS LAUFER MATHEMATICAL SCIENCES INSTITUTE

17 Gauss Way

FALL 2024

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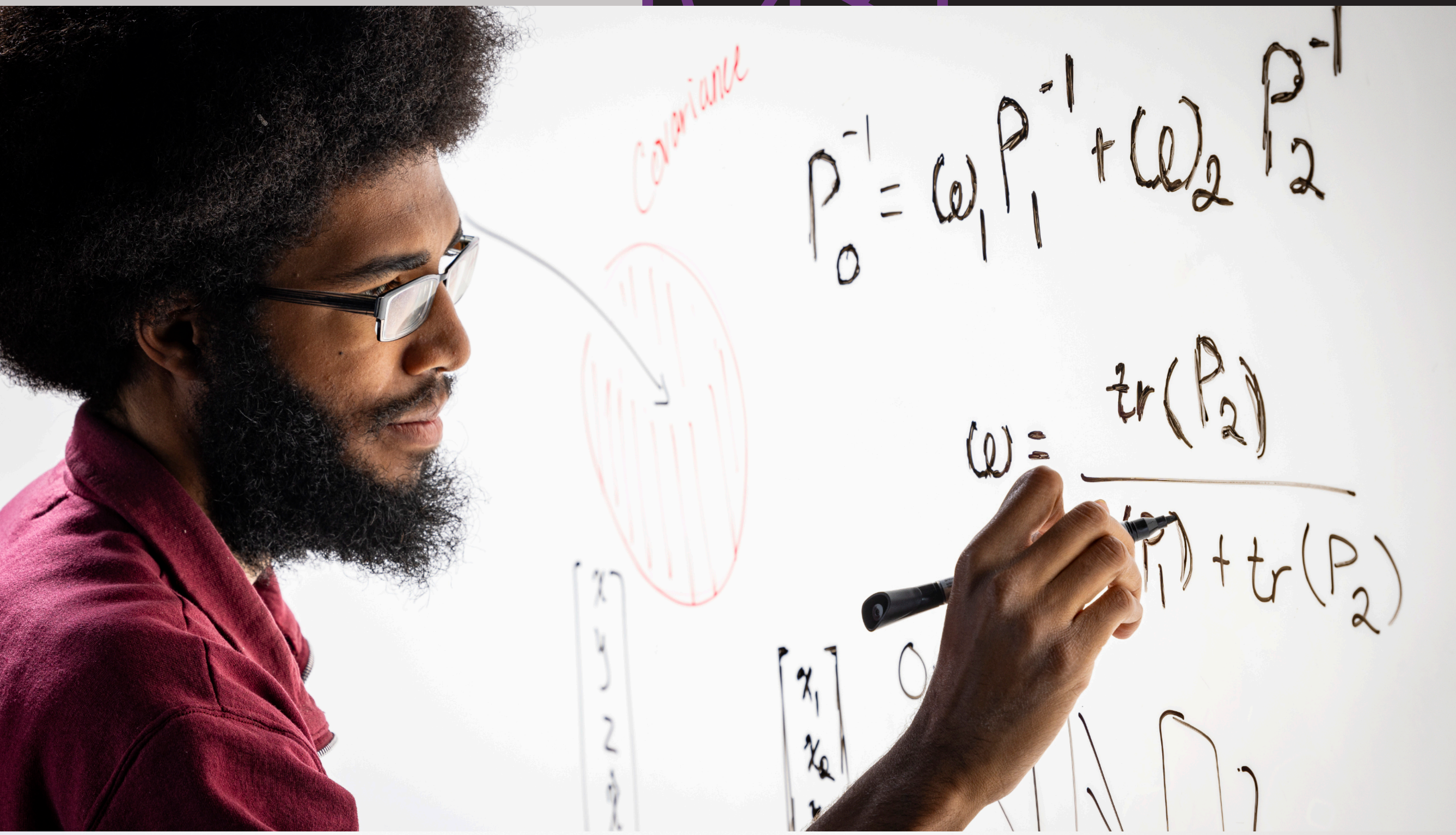

IN THIS ISSUE

SLMATH | # 17 Gauss Way

FALL 2024

On the Cover: Kobe Lawson-Chavanu, an MSRI–UP alumni, is one of the many individuals featured in *Journeys of Black Mathematicians: Creating Pathways* (article on page 22). Photo: Ed Whitman,

Below: Attendees of the SLMath Summer Graduate School on the h-principle, jointly organized by SLMath with RIKEN, in Sendai, Japan (July 1–12, 2024).

Questions and comments regarding *17 Gauss Way* should be directed to **newsletter@slmath.org**

The newsletter archive is available online at **slmath.org/newsletter-archive**

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SLMath, formerly MSRI, has been supported from its origins by the U.S. National Science Foundation, joined by the National Security Agency, over 110 Academic Sponsor Institutions, private foundations, and generous and farsighted individuals.

DIRECTOR'S

Update

Tatiana Toro, Director

SLMath's Journalist in Residence, Siobhan Roberts, gave a public talk on "The Art & Science of Writing About Math" in October.

Over the last year, while engaged in the NSF mathematical sciences institutes grant competition, I had the opportunity to reflect on how SLMath has fulfilled its mandate to serve as a national resource. According to National Center for Science and Engineering Statistics (NCSES) data, there are 199 institutions that have granted a Ph.D. in mathematics since 2008. Of those, 95% have sent participants to an SLMath activity in the same time window. As shown by this data, SLMath has a significant impact on the research landscape of the mathematical community: mathematicians bring their expertise to the Institute and return home energized, having established exciting collaborations. SLMath's impact is a reflection of the Institute's mission, which permeates all of SLMath's programs and activities, as illustrated below.

Welcoming Researchers

This fall, SLMath welcomes researchers in two scientific programs, *New Frontiers in Curvature: Flows, General Relativity, Minimal Submanifolds, and Symmetry* and *Special Geometric Structures and Analysis*. These programs live in the general area of differential geometry, where there have been several fundamental developments in recent years including the solution of the longstanding Willmore conjecture by **Fernando Codá Marques** (Simons Visiting Professor at SLMath) and **André Neves** (Clay Senior Scholar at SLMath). The techniques they developed are playing a central role in these semester's programs. The Ricci flow, introduced by Richard Hamilton, which was used to prove the Poincaré conjecture and geometrization conjecture, is also predominantly featured in many of the seminars this semester. The 2020 program Quantum Symmetries, interrupted by the COVID pandemic, also held a month-long in person reunion at the Institute.

Developing Young Talent

Through the Summer Graduate Schools (SGS), SLMath serves its Academic Sponsors, as well as all US institutions. They provide an invaluable interface between SLMath and the mathematical community. In 2024, SLMath hosted 12 SGS in California as well as in Canada, Switzerland, Japan, and Greece, in addition to MSRI–UP, ADJOINT and Summer Research in Mathematics (SRiM) research groups, and MAY–UP, aimed at freshmen from the Atlanta University Center schools. In particular, the youthful energy of MSRI–UP's cohort of scholars in this year's "Mathematical Endocrinology" program contributed to creating a vibrant, inspiring atmosphere in the building.

Sharing the Power of Mathematics Through Writing

Also starting this past summer, SLMath welcomed Journalist-in-Residence **Siobhan Roberts**, a Canadian science journalist, *New York Times* contributor, and biographer. The Journalist in Residence program is a vehicle to expand the public's awareness of the role the mathematical sciences play in everyday life. The journalist is in residence at SLMath for up to four months; interacts with mathematicians; writes at least three articles about mathematics (none about the Institute); gives a public talk; and leads a professional development workshop about how to effectively communicate scientific information to the public.

In October, Siobhan gave a public talk at SLMath on "The Art & Science of Writing About Math" (pictured above), as well as a series of workshops for members. She recently published an article in the *New York Times* about the documentary film *Counted Out* that was screened at SLMath in July: the article is titled "Why Democracy Lives and Dies By Math" and features a conversation with director Vicki Abeles.

SLMath will host a December professional development seminar on "Large-Scale Problems and Challenges in Mathematical Publishing," organized by **Ilka Agricola** (Philipps-Universität Marburg), a member of the International Mathematical Union (IMU) Committee on Electronic Information and Communication. The seminar will feature remote panelists **Ivan Oransky**, a co-founder of Retraction Watch, Editor in Chief of *The Transmitter*, journalist in residence at NYU, and Special Advisor for Scientific Publishing at the Simons Foundation overseeing support of the arXiv, and **Olaf Teschke** (FIZ Karlsruhe), Managing Editor of zbMATH Open and Chair of the Publication and Electronic Dissemination Committee of the European Mathematical Society.

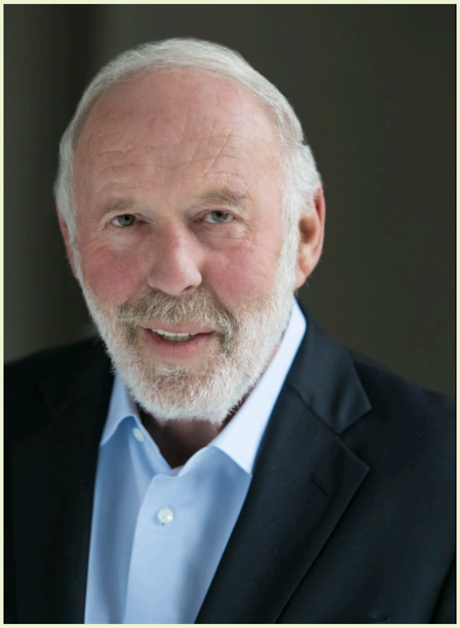
We are also thrilled to announce the Mathical Readathon, an initiative of SLMath's Mathical Books program, which is co-chaired by **John Urschel** (MIT), **Katie Hendrickson** (Code.org), and **Chris**

(Update continues→)

Remembering Dr. James Harris (Jim) Simons (1938–2024)

An award-winning mathematician, a legend in quantitative investing, and a generous philanthropist through the Simons Foundation and many other initiatives, Jim's legacy in the mathematical sciences and the research community at large is immeasurable.

From 1968 to 1978, he chaired the math department at Stony Brook University in New York, and his mathematical breakthroughs during that time are now instrumental to fields such as string theory, topology and condensed matter physics. In 1978, Jim founded what would become Renaissance Technologies, a hedge fund that pioneered quantitative trading and became one of the most profitable investment firms in history. He then turned his focus to making a difference in the world through the Simons Foundation, Simons Foundation International, Math for America, and other philanthropic efforts.



Jim's belief and support of MSRI, now the Simons Laufer Mathematical Sciences Institute, goes back to the very beginning, including his service as a Trustee for 25 years. His work will live on through his mathematical research and the many, many lives and organizations touched by his and his wife Dr. Marilyn Hawrys Simons' determination and passion for advancing the frontiers of mathematics and the basic sciences.

The Simons Foundation has shared reflections on Jim's life and careers at www.simonsfoundation.org/observing. A tribute to Jim Simons will be featured in the January 2025 issue of the *Notices of the American Mathematical Society*, featuring reflections by Tatiana Toro, David Eisenbud, and many other friends and colleagues of Jim over his lifetime.

Nho (Desmos, Public Math). Through the 2024–25 school year, the Readathon invites US educators at public, private, and charter schools in Pre-K through 8th grade to read and share any of the 152 books that have been named a Mathical Award Winner or Honor Book. The books are paired with monthly hands-on math activities for ages 3–13. You can visit page 11 for more about the Readathon and Mathical's recent work.

In Memoriam

I would like to share our sorrow at the recent loss of two remarkable individuals in our mathematics community:

Richard Hamilton (1943–2024). In light of his legacy in the areas of mathematics represented in this semester's scientific programs, we would like to honor Richard's memory and publicly recognize his work and impact. An obituary written by SLMath journalist in residence Siobhan Roberts was featured in the *New York Times* in early December: "Richard Hamilton, Who Helped Solve a Mathematical Mystery, Dies at 81."

Mourning the loss of a visionary: Jim Simons (1938–2024). I wish to express my profound respect and admiration for the incredible life of a warm human being, whose generosity of spirit has impacted many lives around the world, and has transformed the landscape of science forever. See Remembering Jim Simons (above).

Thank You to the Community

We acknowledge the Institute's founders and the support of the NSF as the bedrock of SLMath's success. The Institute is stewarded by its Board of Trustees and endorsed by our generous donors. We are grateful to all in the mathematics community and beyond who have contributed to the Institute's success in supporting collaborative, impactful, and cutting edge mathematics research, developing talent, and inspiring an appreciation of the power, beauty, and joy of mathematics.

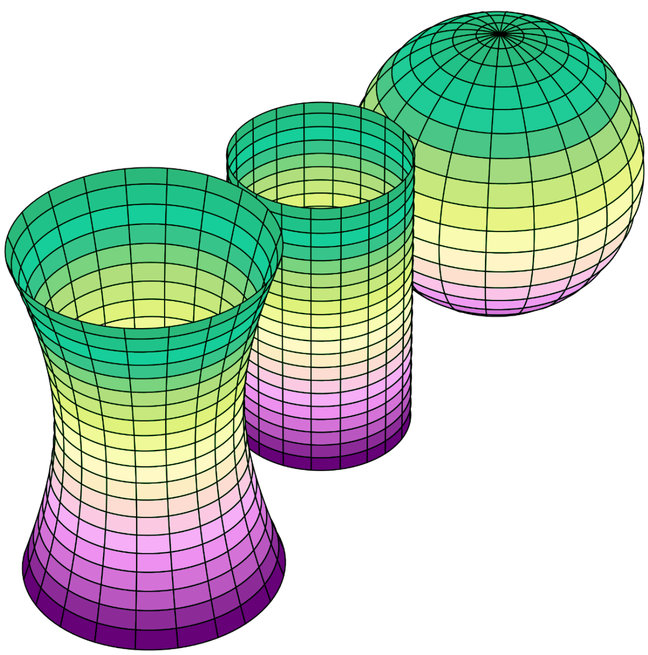
Program Article

Minimal Surfaces and Flows in General Relativity

Ailana Fraser, Lan-Hsuan Huang, Richard Schoen, Catherine Searle, Lu Wang, and Guofang Wei

Since the 19th century, when Riemann introduced the concept of curvature tensors to describe curved spaces in three dimensions or higher, the idea of curvature has become widely applicable in many scientific fields. This concept gained its foundational significance in 1915 when Einstein developed his theory of general relativity, which models our universe as a four-dimensional spacetime governed by the Einstein field equations. These equations describe how matter and energy cause spacetime to curve, and in turn, how this curvature influences the movement of objects like cosmic bodies and light.

Einstein's theory of general relativity has been incredibly successful, not only in enhancing our understanding of everyday phenomena such as accurate timekeeping with atomic clocks and GPS satellite navigation, but also in predicting extraordinary cosmic events, such as gravitational lensing, gravitational waves, and black holes—objects once thought to be purely theoretical.



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Spaces of negative, zero, and positive curvatures.

From a geometric perspective, general relativity has been deeply shaped by advances in Riemannian geometry, and in turn, it has energized further progress and continually generated new challenges within the field. The *New Frontiers in Curvature* program seeks to bridge various areas of geometry, partial differential equations, and relativity, fostering collaboration across these disciplines. Although the program does not aim to cover all aspects of these broad areas, its central focus is to advance our understanding of Riemannian manifolds under geometric constraints, such as curvature lower bounds.

To achieve this, the program is exploring and promoting new connections between cutting-edge techniques, including minimal submanifolds, which are surfaces derived from minimization of area, and geometric flows, which study the evolution of shapes over time. The program centers around four main themes: geometric flows, geometric problems in mathematical relativity, global Riemannian geometry, and minimal submanifolds, each contributing to a deeper understanding of geometry.

In this article, we focus on the geometric aspects of mathematical relativity and a small sample of its interactions with other main areas of the program.

The Role of Riemannian Geometry in Relativity

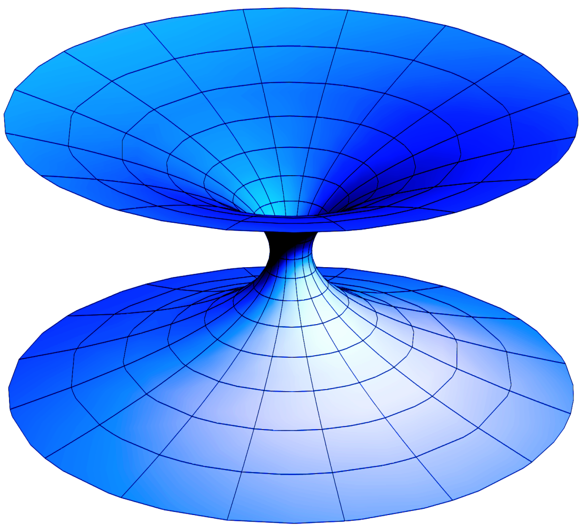
In general relativity, the universe is described as a four-dimensional spacetime, represented as $(N, \mathbf{g})$. Here, N is a smooth, four-dimensional manifold, and $\mathbf{g}$ is a Lorentzian metric of the signature $(-,+,+,+)$, which captures both time and space in a unified framework. One of the key objects in general relativity is the Einstein tensor, $G(\mathbf{g}) = \mathrm{Ric}_{\mathbf{g}} - \frac{1}{2}R_{\mathbf{g}}\mathbf{g}$, where $\mathrm{Ric}_{\mathbf{g}}$ and $R_{\mathbf{g}}$ are the Ricci and scalar curvatures of $\mathbf{g}$, respectively. The core idea of general relativity is that the Einstein tensor is determined by the presence of matter content, and this curvature tells free-falling objects how to move along geodesics. A vacuum spacetime is one with no matter present, meaning that $G(\mathbf{g}) = 0$, which is equivalent to saying that $\mathbf{g}$ has zero Ricci curvature. Minkowski spacetime is flat spacetime, analogous to Euclidean space but in the context of a Lorentzian signature.

Spacetime can be seen as a series of evolving three-dimensional slices, starting from an initial data set. An initial data set (M, g, k) consists of a three-dimensional Riemannian manifold (M, g) with k as the induced second fundamental form. This is called an initial data set because the Einstein tensor $G(\mathbf{g})$, in harmonic gauge, behaves like a wave. According to the Cauchy problem, a vacuum initial data set uniquely determines a local vacuum spacetime. As a submanifold in the spacetime, the Gauss and Codazzi equations impose constraints on this data. Under physical energy conditions, these constraints imply that g has a scalar curvature lower bound, nonnegative in special cases, motivating increased interest in Riemannian manifolds with nonnegative scalar curvature.

The first nontrivial example of a vacuum solution is the Schwarzschild solution, which describes the exterior spacetime surrounding a spherically symmetric, non-rotating, isolated gravitational system with total mass m. The spacetime is $\mathbb{R} \times (\mathbb{R}^3 \setminus B_{2m})$ with the Lorentzian metric

$$\mathbf{g}_m = -\left(1 - \frac{2m}{\rho}\right) dt^2 + \frac{1}{1 - \frac{2m}{\rho}} d\rho^2 + \rho^2 g_{S^2}.$$

When $m > 0$, the Schwarzschild solution describes a black hole with $\rho = 2m$ marking the event horizon. Each constant t-slice is a manifold $\mathbb{R}^3 \setminus B_{2m}$ with the induced metric $g = \frac{1}{1-\frac{2m}{\rho}} d\rho^2 + \rho^2 g_{S^2}$ and zero second fundamental form, providing examples of manifolds with zero scalar curvature. When $m = 0$, it is flat Euclidean space, but for $m > 0$, it gives a complete manifold with a minimal surface at $\rho = 2m$. For $m < 0$, the curvature becomes infinite at $\rho = 2m$, and the metric is incomplete.



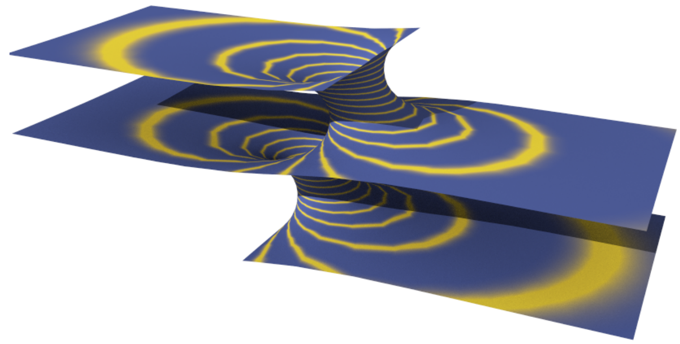
The constant t-slice in the Schwarzschild solution of positive mass (one space dimension suppressed).

The Schwarzschild solution has inspired further investigations into many surprising phenomena, including spacetime singularities, black holes, and the definition of total mass for an isolated gravitating system, the so-called ADM mass. Exploring these concepts leads to notable examples of the applications of minimal surfaces and geometric flows in general relativity, which we will discuss below.

Minimal Surfaces

Let Σ be a closed surface in a three-manifold. The parallel surfaces Σ_t are generated by moving Σ uniformly along the normal direction. The first-order variation of the area $A(t) = \text{Area}(\Sigma_t)$ gives the mean curvature $H = A'(0)$. If $H = 0$, Σ is called a minimal surface.

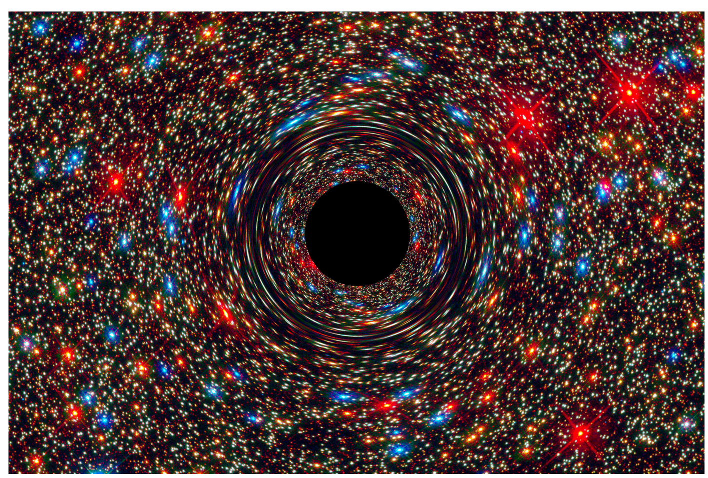
These surfaces have been studied since the 18th century and remain a central area of interest in differential geometry, with applications in mathematics, physics, and engineering. After the advent of general relativity, minimal surfaces found important applications in the field relating to black hole theory.



Three levels of Riemann's minimal surface, rendered with a texture to show that the intersection with horizontal planes is circular. (The original surface mesh is from the computational and scientific graphics laboratory at MSRI).

In his 1965 singularity theorem, Penrose introduced the concept of trapped surfaces and demonstrated that the existence of a closed trapped surface implies a light ray from it will eventually reach a spacetime singularity. His work implied that the singular behavior of black holes persists under perturbations, leading many to believe that black hole formation is a genuine physical phenomenon, earning him a Nobel Prize in 2020.

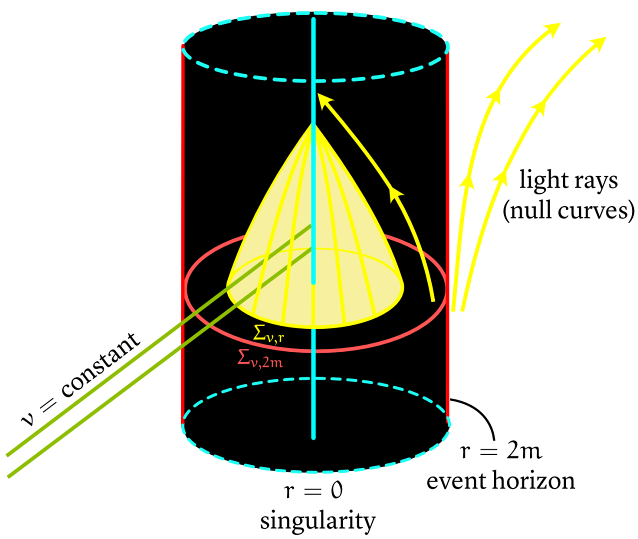
A trapped surface is defined using area variations. Given a 2-surface Σ in spacetime, consider shooting a light ray from each point of Σ to form a surface Σ_t, representing a "shell of light" emanating from Σ. In strong gravity, the area of these outgoing light shells can decrease at each point of Σ, indicating that Σ is a trapped surface. A marginally outer trapped surface (MOTS) is characterized by the first-order variation of the area being zero, similar to a minimal surface. The difference is that the parallel surfaces are now varied in the null direction. However, in special cases, including Schwarzschild spacetime, a MOTS corresponds to a minimal surface in an initial data set.



This computer-simulated image shows a supermassive black hole at the center of a galaxy. The black region represents a snapshot of the event horizon of the black hole. Light from background stars is stretched and distorted due to the strong gravity of the black hole. Image: NASA, ESA, and D. Coe, J. Anderson, and R. van der Marel (STScI).

By investigating the second-order variation of the area, one can further explore the intriguing relationship between scalar curvature bounds and minimal surfaces. A notable achievement is the minimal surface theory introduced by Schoen and Yau in their proof of the positive mass theorem. Asymptotically flat three-manifolds are employed to describe isolated gravitating systems. These manifolds are "modeled" on the constant t-slice of the Schwarzschild solution, meaning that they approximate the geometric properties of the Schwarzschild spacetime at large distances.

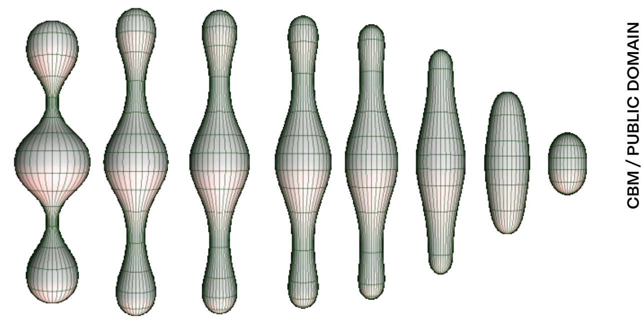
An important special case of the positive mass theorem states that for an asymptotically flat three-manifold with nonnegative scalar curvature, the ADM mass must be nonnegative. A brief overview of the minimal surface proof is as follows: if the ADM mass is negative, a complete minimal surface with a planar end can be constructed, which ultimately contradicts the Gauss–Bonnet theorem. The positive mass theorem is fundamental in general relativity because it implies that Minkowski space, with zero ADM mass, is "stable." If the mass were negative, it would suggest instabilities or exotic phenomena not observed in nature.



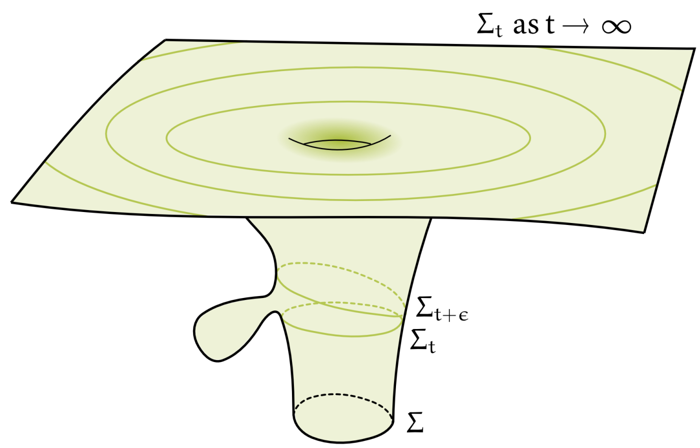
The Schwarzschild metric in the ingoing Eddington–Finkelstein coordinates (one space dimension suppressed). Any light ray that starts at a point where $r < 2m$ will crash into the $r = 0$ singularity. The surfaces $\Sigma_{v,r}$ defined by constant v, r coordinates are trapped surfaces whenever $r < 2m$, and they are MOTS when $r = 2m$. Image: Lan-Hsuan Huang.

Flows

Geometric flows examine how geometric objects evolve to reach their optimal forms. Prominent examples include the Ricci flow and mean curvature flow. A major focus is on understanding the behavior of these flows near singularities using techniques such as blow-up analysis and monotonicity formulas. These methods give rise to self-similar solutions, or solitons, which are crucial for singularity models and have connections to other geometric subjects. For instance, self-similar solutions of mean curvature flow are minimal submanifolds under specific metrics, and Ricci solitons generalize Einstein manifolds.



Several stages of the Ricci flow on a 2D manifold.



In an asymptotically flat 3-manifold (one dimension suppressed), the IMCF evolves from the surface Σ toward large coordinate spheres. Image: Lan-Hsuan Huang.

Geometric flows have already demonstrated their significance in general relativity. For example, the Ricci flow offers an alternative proof of the positive mass theorem mentioned above. Another notable flow, called the inverse mean curvature flow (IMCF), is an important tool for proving the Penrose inequality. A one-parameter family of surfaces in a Riemannian manifold, $t \mapsto \Sigma_t$ evolves under IMCF if it satisfies the equation

$$\frac{\partial}{\partial t}\mathbf{x} = H^{-1}\nu \quad \text{for } \mathbf{x} \in \Sigma_t,$$

where $H > 0$ is the mean curvature of the surface and ν is the outward unit normal vector to the surface. In an asymptotically flat three-manifold the Geroch monotonicity formula states that, under the IMCF, the Hawking mass is non-decreasing, where the Hawking mass is given by

$$m_H(\Sigma_t) = \frac{1}{16\pi}\sqrt{\frac{A(\Sigma_t)}{16\pi}}\left(16\pi - \int_{\Sigma_t} H^2\, dA\right).$$

At a minimal surface boundary, the Hawking mass becomes $\sqrt{A(\Sigma_t)/16\pi}$, measuring the mass of the black hole enclosed by the minimal surface. As $t \to \infty$, the Hawking mass converges to

FOCUS *on the Scientist* Regina Rotman


AARON FAGERSTROM

Regina Rotman is a research professor in the *New Frontiers in Curvature* program. Regina earned her Ph.D. at SUNY at Stonybrook under the direction of Detlef Gromoll in 1998. After graduation, she was an NSF postdoctoral fellow and a visiting professor at the Courant Institute and the University of Toronto. She joined the faculty at Penn State in 2003, and in 2005 she returned to the University of Toronto, where she has been a full professor since 2013. Regina has made outstanding contributions to the area of quantitative geometry, where the main objective is to study quantitative aspects of solutions to variational problems on Riemannian manifolds whose existence has been previously established via methods of algebraic topology.

Examples of the types of existence theorems she tries to make quantitative are the work of A. Fet and L. Lusternik demonstrating that on any closed Riemannian manifold there exists a periodic geodesic, as well as a famous theorem of J.P. Serre proving that for any pair of points on a closed Riemannian manifold there exist infinitely many geodesics connecting these points. A natural question is whether one can bound the length of these geodesics in terms of the geometric parameters of the manifold.

As part of her long list of contributions, Regina established the best currently known area upper bound for the length of a shortest closed geodesic on a Riemannian 2-sphere (with Liokumovich and Nabutovsky); answered a 20-year old question of Frankel and Katz about an optimal contraction of the boundary of a Riemannian 2-disk; found a diameter upper bound for the length of a shortest

Regina has a long list of outstanding contributions to the area of quantitative geometry.

geodesic loop at each point of a closed Riemannian manifold; found curvature-free upper bounds for the area of a smallest minimal surface (with Nabutovsky); provided an answer to Gromov's question about the length of a shortest periodic geodesic on closed Riemannian manifolds with positive scalar curvature in dimension 3 (with Liokumovich and Maximo); and established a quantitative version of Serre's existence theorem (with Nabutovsky).

In addition to being a prolific researcher, Regina is also focused on mentorship of the next generation of mathematicians. She has supervised over 34 people and at all levels: undergraduate, masters, Ph.D., and postdoctoral. She says that she is "especially proud of her students' successes" and "feels privileged to have had the opportunity" to work with them.

— Catherine Searle

the ADM mass of the isolated system. Thus, monotonicity plays a key role in Huisken and Ilmanen's proof of the Riemannian Penrose inequality, which states that the total mass of an isolated gravitational system is no less than the mass contained in its black hole.

Other Aspects of the Program

In this article, we have explored some influential applications of minimal surfaces and geometric flows in solving fundamental problems in general relativity. Although we have only focused on three dimensions, there is strong motivation coming from both mathematics and theoretical physics to extend results to higher dimensions.

Another aspect of our program is understanding spaces with weak regularity. Just as limits of smooth functions may not remain smooth, limits of Riemannian manifolds can lack smooth structures. On the Riemannian side, this motivates the investigation of non-smooth spaces with lower curvature bounds. For instance, if a sequence of manifolds has uniform lower bounds on sectional curvature, the limits are Alexandrov spaces. If they have lower bounds on Ricci curvature, they become Ricci limit spaces. Both types of limit spaces have been studied very actively in recent decades.

On the Lorentzian side, as alluded to earlier, it is also essential to understand Lorentzian spacetimes with singularities, as they are believed to be a common feature according to our understanding of black holes, with singularities thought to lie hidden behind their event horizons. Results from Riemannian geometry are continually finding applications in the study of singular Lorentzian geometry, an important field with a great deal remaining to explore.

Graduate Fellows / FALL 2024

Isabel Beach is the SLMath graduate fellow in the *New Frontiers in Curvature* program. They are currently a fifth year Ph.D. candidate at the University of Toronto, supervised by Regina Rotman. They previously earned a bachelor's degree in mathematics at the University of Toronto. Isabel studies quantitative geometry, focusing on the quantitative behavior of geodesics and minimal submanifolds.

Jihye Lee is the Kristin E. Lauter graduate fellow in the *New Frontiers in Curvature* program. She is currently a Ph.D. candidate at the University of California, Santa Barbara under the supervision of Guofang Wei. She is interested in geometric analysis and comparison geometry. Her research focuses on geometric inequalities under various curvature assumptions. In particular, she investigates logarithmic Sobolev inequality with intermediate Ricci curvature condition and isoperimetric profile comparison with integral Ricci curvature. *(Photo: Aaron Fagerstrom)*

Henrik Naujoks is the Stephen Della Pietra graduate fellow in the *Special Geometric Structures and Analysis* program. He is currently a second-year Ph.D. student under the supervision of Ilka Agricola at Philipps University in Marburg, Germany. He previously completed both his bachelor's and master's degrees in physics at the same institution. This background has fostered his interest in fields at the intersection of mathematics and physics, such as gauge theory, spin geometry, and geometric field theories. *(Photo: Aaron Fagerstrom)*

*Graduate fellowships support current graduate students to take part in our research programs, thanks to the support of **SLMath individual donors** and **private foundations**. These fellowships allow graduate students to receive financial support so that they can remain in residence at SLMath for the entire semester with their advisor, fully integrated into the semester's research program.*

Named Positions / FALL 2024

Chern, Clay, Eisenbud, Simons Professors

Claudio Arezzo, Abdus Salam International Center for Theoretical Physics
Eleonora Di Nezza, Sorbonne Université & École Normale Supérieure
Ailana Fraser, University of British Columbia
Bo Guan, Ohio State University
Ursula Hamenstädt, Universität Bonn
Mark Haskins, Duke University
Jason Lotay, University of Oxford
Fernando Codá Marques, Princeton University
Tristan Rivière, ETH Zürich
Richard Schoen, University of California, Irvine
Peter Topping, University of Warwick
Guofang Wei, University of California, Santa Barbara

Named Postdoctoral Fellows

Chern: Alec Payne, Duke University
Gamelin: Adrian Chun-Pong Chu, The University of Chicago
Huneke: Giada Franz, Massachusetts Institute of Technology
Uhlenbeck: Shuli Chen, Stanford University
Viterbi: Qin Deng, Massachusetts Institute of Technology

Named Program Associates

Kristin E. Lauter: Jihye Lee, University of California, Santa Barbara
Stephen Della Pietra: Henrik Naujoks, Philipps-Universität Marburg
SLMath: Isabel Beach, University of Toronto

SLMath is grateful for the generous support that comes from endowments and annual gifts that support faculty and postdoc members of its programs each semester.

SLMath — An Initial Perspective

Juan C. Meza, Associate Director

I'm delighted to be able to share a few thoughts as the newly appointed Associate Director of SLMath. I started in August, but it's already been an exciting and eye-opening experience.

When I served as Division Director for the NSF's Division of Mathematical Sciences, I was of course familiar with all the NSF Mathematical Sciences Institutes and the wonderful work they have done and continue to do. The institutes serve a critical role in building and maintaining the health of the nation's mathematical research community. Through their efforts, they help the community come together and work on some of the most challenging research problems at the frontiers of the mathematical and statistical sciences. The institutes also serve an important role in training future generations of mathematical scientists.

Serving as Associate Director (even for a brief time) has deepened my appreciation for what institutes such as SLMath contribute to the mathematical community and the nation's research infrastructure more broadly. For instance, I was surprised to hear that out of the 199 Math Ph.D. granting departments in the U.S., 186 (95%) have sent someone to an activity at SLMath. This is an incredible testament to the impact that SLMath has on the entire US mathematics community.

Developing Collaborations and Careers

This year at SLMath, there are four exciting programs at the leading edge of several areas in geometric analysis, extremal combinatorics, and probability and statistics. What has impressed me most is not just the deep technical expertise of all the participants, but the collaborations that are already starting to develop across this semester's two programs.

In addition, I've seen a deep commitment to mentoring the next generation, especially the postdoctoral fellows and early-career researchers. I've been able to meet many of the participants through the social events, and everyone I have met seems truly excited to be here.

As part of our professional development efforts, I've been working with two postdocs (Anna Skorobogatova and Freid Tong) in re-starting a career development seminar. The first seminar was on how to write a grant, in which I gave a talk that was followed by a panel of four senior mathematicians who are part of the programs. In the second seminar, we hosted four panelists who spoke on the topic of how to write research, teaching, and diversity statements.

Both seminars were well attended with great questions from the postdocs in the audience. It was also heartening to see that it's not just the early career researchers who attend but also the senior researchers who took time away from their research to attend and provide additional advice based on their personal experiences. An interesting outcome of the additional mentoring provided is that 7% (22) of the NSF DMS CAREER awards made over the past five years were awarded to former SLMath postdocs.

Behind the Scenes with the Advisory Committees

Another aspect of the role that I've enjoyed is working with the various advisory committees in developing new programs. The Scientific Advisory Committee is outstanding in selecting from among the many truly excellent proposals we receive. They have a daunting task, which they take very seriously.

I've also had the privilege of working with the Broadening Participation Advisory Committee. Their commitment to ensuring that all SLMath's activities are as inclusive as possible is truly amazing. Having served on the committee previously, I understand the challenges they face and am impressed by all their work in making our programs successful, diverse, and inclusive.

Juan Meza is SLMath Associate Director for academic year 2024–25, and Professor of Applied Mathematics at the University of California, Merced.

SGS and Academic Sponsors

Finally, I have to say a few words about the Summer Graduate Schools (SGS) and the work of the Committee of Academic Sponsors. The Summer Graduate Schools play an important role in training the next generation of mathematicians. Here again, this SLMath activity is a great example of serving as a national resource. The Summer Graduate Schools draw participants from across the entire U.S. with roughly equal numbers from states in the Northeast, Midwest, South and West. Overall, about 17% of the SGS students come from NSF Established Program to Stimulate Competitive Research (EPSCoR) states, which are the U.S. states which receive the lowest ratio of federal research and development funding. This is a testament to the great work that the Academic Sponsors do in nominating students.

I'll close with a quote from Marcus du Sautoy: "Mathematics has beauty and romance. It's not a boring place to be, the mathematical world. It's an extraordinary place; it's worth spending time there." Likewise, I'm sure that in this coming year, SLMath will prove to be an extraordinary place to be, and I encourage you to come visit us and spend some time here.

Mathical Launches Readathon for U.S. Pre-K–8 Classrooms

SLMath's Mathical Books program selects, promotes, and distributes math-inspiring fiction and nonfiction for children ages 2–18 in partnership with the Children's Book Council, leading educator organizations, and nonprofit organizations serving low-income youth.

Readathon Runs Through Next June

In the 2024–25 school year, Mathical has launched a pilot Readathon program for Pre-K through 8th grade classrooms across the U.S., inviting teachers and students to read and share the 150+ inspiring fiction and nonfiction stories which have been awarded the Mathical Book Prize since 2015.

Through June 2025, schools are invited to join in at mathicalbooks.org/readathon; there is also a monthly Mini News celebrating reading milestones and sharing activities, including fun new resources created with Mathical winning author-illustrators Lalena Fisher (*Friends Beyond Measure: A Story Told in Infographics*) and Don Tate (*Jerry Changed the Game! How Engineer Jerry Lawson Revolutionized Video Games Forever*).

Step right up for the power of Mathical's free books and Readathon (with SLMath's Jenn Murawski and Kirsten Bohl).

Conference for Elementary Math & Literacy

In June 2024, Mathical joined the National Council of Teachers of English (NCTE) and the National Council of Teachers of Mathematics (NCTM) for their first Joint Conference for Elementary Mathematics and Literacy, held in New Orleans, LA.

Mathical presented educator sharing sessions for conference attendees, and in partnership with publisher Science, Naturally!, SLMath distributed 1,500 copies of the Mathical winning books *Women in Engineering* by Mary Wissinger and *Leonardo da Vinci Gets A Do-Over* by Mark P. Friedlander, Jr., to conference attendees and Title I schools serving low-income communities in the region.

Conference attendees also enjoyed meeting Mathical authors Grace Lin (*What Will Fit?* and *Up to My Knees!*), Don Tate (*Jerry Changed the Game!*), and Danica McKellar (*Goodnight, Numbers!*), who gave the event's opening and keynote addresses.

JASON MAZE

Brady Haran presents at the MSRI/SLMath 40th Anniversary Symposium.

Numberphile's Brady Haran Awarded Christopher Zeeman Medal

In September 2024, the Councils of the Institute of Mathematics and its Applications (IMA) and the London Mathematical Society announced Brady Haran, creator of YouTube's Numberphile channel, as the 2024 Christopher Zeeman Medal awardee. The Christopher Zeeman Medal is the UK award dedicated to recognizing excellence in the communication of mathematics.

The award citation explains that, "since 2011, [Numberphile's] videos have been watched nearly 700 million times and inspired many young viewers to pursue a career in mathematics (and caused many older viewers to wish they had!). They are among the most viewed mathematics content in the world."

Brady recognized his longtime collaborator, artist, animator, and editor Pete McPartlan as part of Numberphile's success in sharing mathematical content with hundreds of millions of viewers worldwide.

You can view the full citation at www.lms.ac.uk/news/brady-haran-2024-christopher-zeeman-medal, and read Brady's blog post reflecting on the award at www.bradyharanblog.com/blog/the-zeeman-medal.

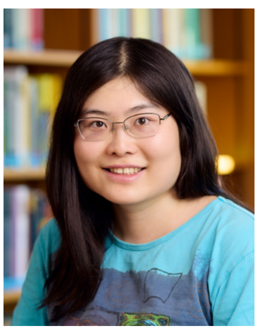
Distinguished Postdoctoral Fellowships / FALL 2024

GAMELIN

Adrian Chun-Pong Chu is the Gamelin postdoctoral fellow in the *New Frontiers in Curvature* program. He obtained his bachelor's degree at the Chinese University of Hong Kong and his Ph.D. at the University of Chicago under André Neves. Following SLMath, he will be a postdoc at Cornell University in 2025. Adrian's research has focused on minimal surfaces, min-max theory, and mean curvature flow. From a broader perspective, these topics arise from studying the space of all hypersurfaces in a given Riemannian manifold, and the area functional defined on this space.

The Gamelin postdoctoral fellowship was created in 2014 by Dr. Ted Gamelin, Emeritus Professor of the UCLA Department of Mathematics. The Gamelin fellowship emphasizes the important role that research mathematicians play in the discourse of K–12 education.

UHLENBECK

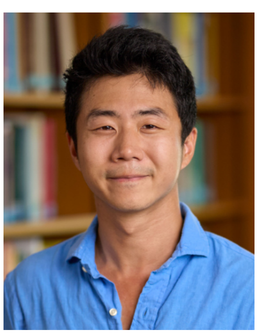


AARON FAGERSTROM

Shuli Chen is the Uhlenbeck postdoctoral fellow in the *New Frontiers in Curvature* program. She attended Cornell University as an undergraduate and then received her Ph.D. in 2024 from Stanford University under the guidance of Otis Chodosh. After the SLMath program, she will join the University of Chicago Department of Mathematics as an L.E. Dickson Instructor. Shuli's research mainly focuses on the stability of minimal surfaces and scalar curvature related problems. She has worked on characterizing stable minimal submanifolds, computing or bounding the Morse index of minimal hypersurfaces, and establishing topological obstructions to positive scalar curvature metrics.

The Uhlenbeck fellowship was established by an anonymous donor in honor of Karen Uhlenbeck, a distinguished mathematician and former MSRI trustee. She is a member of the National Academy of Sciences and a recipient of the 2019 Abel Prize, the AMS Leroy P. Steele Prize, and a MacArthur "Genius" Fellowship.

VITERBI



AARON FAGERSTROM

Qin Deng is the Viterbi postdoctoral fellow in the *New Frontiers in Curvature* program. He obtained his Ph.D. in 2021 from the University of Toronto under the supervision of Vitali Kapovitch. Before coming to SLMath, he was a C.L.E. Moore Instructor at MIT, working with Tobias Colding. Qin is interested in Riemannian geometry, metric geometry, and geometric analysis. Specifically, his research focuses on the geometric properties and structure theory of singular spaces with sectional and Ricci curvature bounded from below, namely Alexandrov and RCD spaces. Some of his results include the non-branching property of geodesics in RCD spaces and a rigidity theorem for Alexandrov spaces with boundary of maximal size.

The Viterbi postdoctoral fellowship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.

Distinguished Postdoctoral Fellowships / FALL 2024

CHERN

AARON FAGERSTROM

Alec Payne is this year's Chern postdoctoral fellow and is a member of both the *New Frontiers in Curvature* and the *Special Geometric Structures and Analysis* programs. Alec received his Ph.D. in 2021 from New York University under the direction of Bruce Kleiner. Before coming to SLMath, he was a Phillip Griffiths Assistant Research Professor at Duke University. Alec has broad research interests spanning geometric flows and special geometric structures. One of the geometric flows he studies is mean curvature flow. He has constructed new special solutions to this flow and has analyzed weak solutions that continue the flow through singularities. Alec also works on G_2-geometry and G_2-Laplacian flow. He has studied the behavior of G_2-structures under curvature restrictions, including the uniqueness and asymptotics of solitons for G_2-Laplacian flow. Alec is also interested in the classical topic of isometric deformations of surfaces and is working on constructing new examples of flexible surfaces in $\mathbb{R}^3$.

The Chern fellowship was established in 2022 by the S.S. Chern Foundation for Mathematical Research through the generosity of S.S. Chern's family. Shiing-Shen Chern was an outstanding contributor to research in differential geometry and was one of the three founders of MSRI, serving as its first director.

HUNEKE

Giada Franz is the Huneke postdoctoral fellow in the *Special Geometric Structures and Analysis* program. She studied at the University of Pisa and Scuola Normale Superiore and received her Ph.D. in Mathematics in 2022 from ETH Zürich under the supervision of Alessandro Carlotto. She then spent two years as a C.L.E. Moore Instructor at MIT and, after this semester's program, she will join CNRS at Gustave Eiffel University. Giada's research interests are in geometric analysis and differential geometry, with a particular focus on the study of minimal surfaces. She has constructed new examples of free boundary minimal surfaces using variational methods and studied the relationship between the topology and the analytic properties of these objects.

The Huneke postdoctoral fellowship is funded by a generous endowment from Professor Craig Huneke, who is internationally recognized for his work in commutative algebra and algebraic geometry.

Art at 17 Gauss Way: Lynn Glaser

SLMath's Fall 2024 art exhibit, Excavation, features paintings by Berkeley artist **Lynn Glaser**, curated by architect William Glass. Glaser's early training in abstraction was in music and dance, and she studied piano from childhood and earned keyboard and music history degrees at university. As an adult, Glaser returned to UC Berkeley to study painting. "My paintings are concerned with a feeling of the openness of large spaces, whether in nature or one's imagination. I hope the viewer also senses a passage of time, by the pace at which one looks at a work and the excavation of paint layers in the work itself."

Over the years, Glaser has experimented with three dimensional painting in the form of freestanding wooden obelisks, as well as with wood panels on which one can "excavate" by sanding through layers of paint. Recently, she has returned to stretched and unstretched linen as well. See lynnglaser.com.

PHOTO SARAH BETH STUDIOS

Untitled (Papageno), 40x50", oil on board, 2019, by Lynn Glaser.

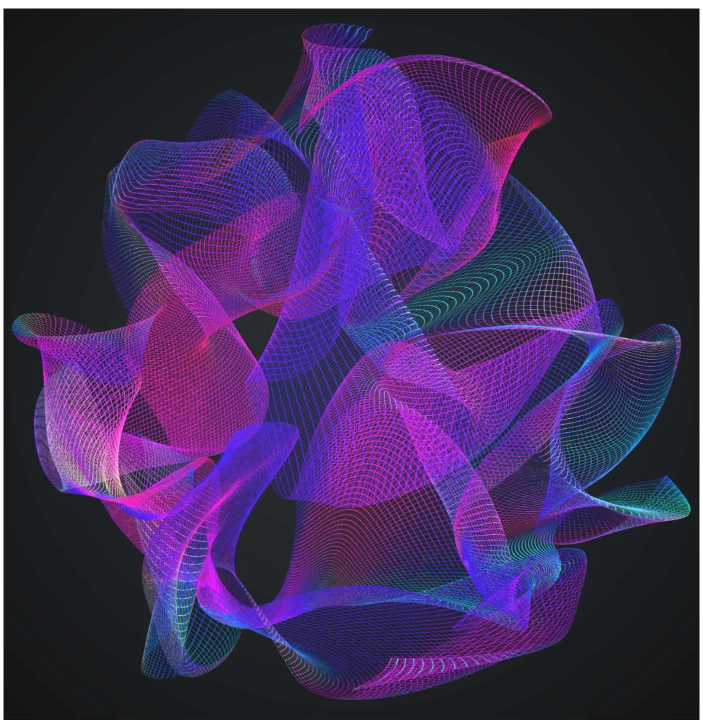
Program Article

Special Geometric Structures and Analysis

Jason Lotay

Geometry lies at the heart of mathematics, going back to the very earliest beginning of the subject and continuing right through to cutting-edge research today. A relatively recent realization is that some types of geometry are more special than others.

The key to this understanding is the central role played by analysis in geometry: that is, the use of calculus techniques. More specifically, it has proven invaluable to formulate many important geometric problems using partial differential equations (like the equation that describes how heat dissipates around a room, known, unsurprisingly, as the heat equation). These partial differential equations are typically second order: to get a sense of this, acceleration is generally viewed as a second order quantity for an object's motion, whereas velocity is first order.



VCHALUP / ADOBE STOCK

A fibration of a Calabi–Yau manifold, which provides important examples of special geometric structures.

Most special geometric problems have the property that they involve first order partial differential equations that imply the more general second order ones. This is both surprising and powerful, unlocking new tools and techniques, yielding connections to other areas of mathematics (including close links to this semester's other program, *New Frontiers in Curvature*) as well as theoretical physics, and providing novel avenues of investigation. It is these special geometric structures, and the related analysis, that form the core of study in this semester's program on *Special Geometric Structures and Analysis*. Some key questions that are being explored in the program include:

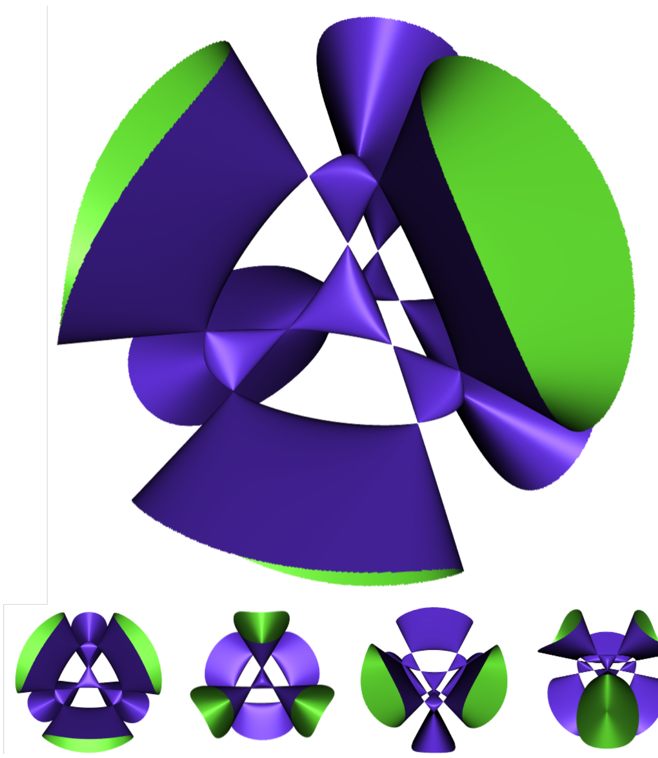
- Do certain special geometric structures exist? Can we determine when they exist?
- Can we understand the families of such structures? Can we classify them?
- How do we find them? How do we overcome the analytic obstacles in known methods for trying to find them?
- What are their properties? Can we understand the singularities (that is, non-smooth parts) of these structures?
- What is the relationship between the various types of special geometric structures? Can we leverage this to further our understanding of their geometry and analysis?

Special Holonomy

To provide a first, concrete example of a special geometric structure and its associated analysis, we want to think about curvature. If one wants to say that a certain average notion of curvature, known as Ricci curvature, is constant, this is equivalent to a second order partial differential equation on the Riemannian metric, which is the object that allows us to define curvature on smooth objects. Geometries where we can do this are called Einstein manifolds, since this condition is an analogue of Einstein's vacuum field equations for gravity from general relativity (and the constant we prescribe corresponds to the cosmological constant in physics). Einstein manifolds are fascinating objects of considerable interest in maths and physics, but they are notoriously hard to find and very mysterious. One particular interest of this program are the Einstein manifolds where the (cosmological) constant is zero: these are the Ricci-flat manifolds.

It is here that special geometric structures enter the story. When one has a Riemannian metric, one can look at how tangent vectors to the space transform as one performs parallel transport around loops: these transformations form a group called the holonomy group. The possible nontrivial holonomy groups were classified in the 1950s by Berger, and on this list there are several special holonomy groups which lead to Ricci-flat metrics: these come in two infinite families (Calabi–Yau and hyperkähler) and two exceptional cases in dimensions 7 and 8 (called G_2 and Spin(7) respectively). The fundamental point is that to find a manifold with a special holonomy group amounts to solving a first order partial differential equation, which implies the second order Ricci-flat condition. In fact, using special holonomy is the only mechanism we know for producing nontrivial compact Ricci-flat manifolds.

Berger's list, mentioned above, only tells you that special holonomy manifolds can exist, not that they actually occur. Their existence, particularly in the compact case, followed from a series of breakthroughs including Yau's solution of the Calabi conjecture (which led to his award of the Fields Medal) and Joyce's construction of the first compact G_2 and Spin(7) manifolds (by modifying the well-known Kummer construction for K3 surfaces, which are the simplest nontrivial examples of Calabi–Yau and hyperkähler manifolds).



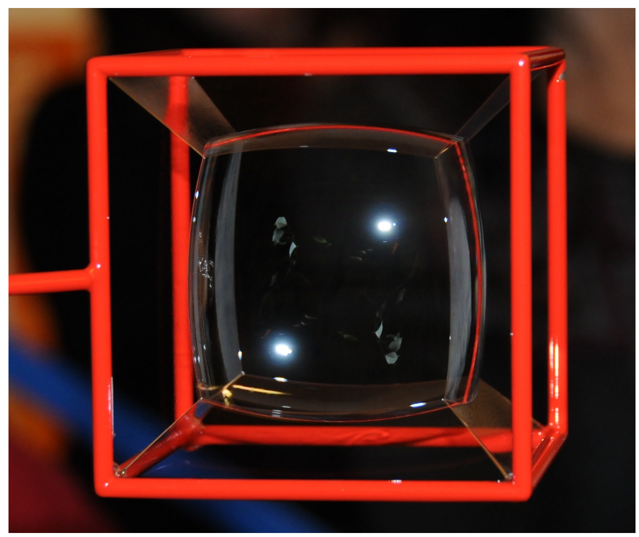
Kummer K3 surface — an important example of a manifold with special holonomy.

One key aspect of this program is to study the geometry and analysis of special holonomy manifolds, as well as related geometries. Of particular interest are the singularities of these spaces: places where they fail to be smooth. Such singular special holonomy spaces are not only of mathematical interest, but naturally arise in the context of theoretical physics in string theory and M-theory.

Another exciting aspect of special holonomy, which is also of primary interest in the program, is the use of geometric flows. Geometric flows are nonlinear analogues of the heat equation mentioned above: they are evolution equations for geometric objects which seek to find an optimal representative as their ultimate objective. This will help to study both the existence question and the classification problem for special holonomy spaces. However, geometric flows do not always work in a straightforward manner: they encounter singularities. These singularities, just as above, are where the flowing object stops being smooth. At this point it is then unclear how to continue the flow. It is these singularities, including how they form, what they look like, and methods for continuing past or around them, that also form an important aspect of the study of geometric flows. In this part of the program, the geometric flows of interest are the Kähler–Ricci flow, as well as other related flows in complex geometry, (which can lead to Calabi–Yau metrics, for example) and the G_2-Laplacian flow introduced by Bryant, which seeks metrics with exceptional holonomy G_2.

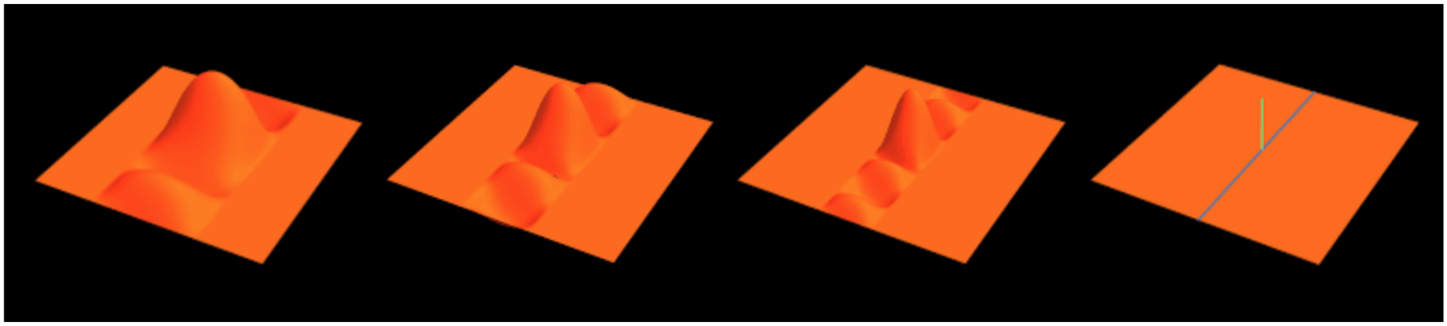
Calibrated Geometry

Closely tied to special holonomy is a special type of submanifold theory: the study of lower-dimensional objects that live inside the higher-dimensional ambient space. One example of a submanifold is the equator, a line sitting on the two-dimensional surface of the Earth. This special submanifold theory is called calibrated geometry, which leads to submanifolds which minimize their volume (or area) amongst all competitors. The key example is that of soap films, which minimize their surface area. Again, the calibrated condition is a first order partial differential equation, which implies the second order condition of being a critical point for volume (known as minimal submanifolds).



A soap film in a cube forming a line of singularities.

Minimizing volume means that calibrated submanifolds provide natural examples of optimal or canonical representatives, which would hope to lead to solving classification problems. Moreover, just like the shortest curves (or geodesics) on a surface encode key aspects of the geometry of the surface, it is hoped that studying calibrated submanifolds leads to uncovering subtle new information, such as invariants, of the ambient special holonomy spaces. Calibrated submanifolds are also believed to help explain the mysterious phenomenon known as mirror symmetry for Calabi–Yau manifolds, which arose first in theoretical physics through the study of calibrated fibrations where the fibres are called special Lagrangian. Calibrated fibrations also play a key role in



Instantons bubbling along a line resulting in a singular instanton. Image Daniel Platt.

other special holonomy manifolds, such as G_2 and Spin(7) manifolds, where the calibrated submanifolds are called associative and coassociative in the G_2 case, and Cayley in the Spin(7) case.

A fundamental issue when studying calibrated geometry is the presence of singularities. Just as can be seen for soap films, these occur naturally, and also need not just be isolated points. The analysis of such geometric phenomena is challenging and this problem forms a central part of the study in this semester's program, both in this particular topic and more generally across special geometric structures.

Finding calibrated submanifolds outside the realm of complex geometry is very difficult. In general there is no clear method for addressing this issue. However, for special Lagrangian submanifolds in Calabi–Yau manifolds, there are additional techniques. These include a geometric flow called Lagrangian mean curvature flow. As before, singularities are of fundamental importance, but now additional insight is given by inspirational conjectures due to Thomas–Yau and Joyce, which suggest relationships between these singularities and stability conditions, including so-called Bridgeland stability conditions on Fukaya categories. This yields fascinating links between differential geometry, geometric analysis, and symplectic topology.

Gauge Theory

The final special geometric structures we shall discuss form another important class of objects in the context of special holonomy: so-called instantons. Instantons are part of the mathematical area of gauge theory, which originated in physics, specifically electromagnetism (including Maxwell's equations and magnetic monopoles) and particle physics (such as Yang–Mills theory). Instantons, like calibrated submanifolds, are minimizers, this time of the Yang–Mills energy. Again, they are defined by a first order partial differential equation which implies the second order condition of being a Yang–Mills connection (a critical point of the Yang–Mills energy).

Inspired by the success of Donaldson's ground-breaking work on instantons on smooth 4-manifolds, which led to his award of the Field Medal, and the proposals of Donaldson–Thomas for gauge theory in higher dimensions, one hopes to be able to use instantons to reveal new information about special holonomy spaces, for example through the potential construction of invariants. This seems related to the discussion we had above regarding calibrated submanifolds: this is not a coincidence. In fact, the link between instantons and calibrated submanifolds goes further, with a form of duality arising between them, which is one of the areas of study in this semester's program. This duality arises through so-called bubbling, where families of instantons concentrate along a calibrated submanifold, yielding an instanton with singularities at the end of the bubbling process. These singularities form a significant obstacle to furthering our understanding of gauge theory in higher dimensions, and again the fact that the singularities are not isolated, but can occur in families, leads to complicated analysis and rich geometry.

Conclusion

There are many crucial aspects of the geometry and analysis of special geometric structures which are recurrent themes throughout the various topics of interest in the program. These themes, and the links between the range of special geometric structures, will be studied throughout the Fall semester here at SLMath. This will undoubtedly lead to new insights on key questions in the field and breakthroughs in the subject in the future.

FOCUS *on the Scientist* Tristan Rivière

Tristan Rivière, an Eisenbud Professor in this semester's *Special Geometric Structures and Analysis* program, has made outstanding contributions to geometric analysis. Recognitions of his contributions include his invited talk at the 2002 ICM in Beijing and his award of the 2003 Stampacchia medal.

Tristan spent a decade as a CNRS researcher in France and held a visiting professorship at the Courant Institute in 1999–2000. He has been a full professor at ETH Zurich since 2003, and between 2009 and 2019 has also served as FIM director, with over 50 conferences organized. He had the privilege to speak about the work of Louis Nirenberg on the occasion of Nirenberg's 2015 Abel Prize award. Tristan has now supervised over 20 Ph.D. students.

Tristan works on nonlinear PDEs and the calculus of variations, and their roles in physics and geometry. Tristan's work has elucidated many aspects of the regularity theory of nonlinear elliptic systems. To give one instance, it had long been recognized that the regularity theory for elliptic systems (as opposed to scalar equations), where the maximum principle no longer plays a decisive role, is much more delicate than for scalar elliptic PDEs. Past generations of analysts and geometric analysts had already made important contributions to this area (including such renowned figures as Charles Morrey, Ennio De Giorgi and Karen Uhlenbeck), but in many cases the optimal regularity of solutions remained unclear.

Tristan's first result on the lack of regularity for weakly harmonic maps put a stop to the search for a general regularity theory for elliptic systems with natural growth. Later on, one of Tristan's key insights has been the recognition of the power and ubiquity of conservation laws hidden within a large class of PDEs and variational problems and how such conservation laws provide a crucial new tool and unifying framework for proving optimal regularity results for such nonlinear vector-valued problems — for example, weakly harmonic maps, Willmore surfaces, and prescribed mean curvature surfaces.

Tristan's work has been deeply influenced by a variety of renowned figures in the field. Early in his career, Haïm Brezis (who served as an unofficial Ph.D. mentor) made a strong impression, especially with his ability to distill matters to their essential ingredients and to bring the community's focus to these issues through his beautiful lectures. Another early deep and lasting influence was Karen Uhlenbeck's work on harmonic maps and Yang–Mills connections.

Tristan's career has undergone several phase transitions.

Tristan's career has undergone several phase transitions. His interactions with Fang-Hua Lin opened his eyes to the realm of possibilities available when one feels free to use the whole arsenal of modern analytic techniques. Together with his work with Robert Hardt this led him to deepen his interest in regularity questions that move beyond the nonlinear PDE world to also encompass such questions in the context of geometric measure theory (GMT). Gang Tian shone a light for him on how geometry could be a source of beautiful and delicate hard analysis and GMT questions. For instance, their joint work on the regularity theory for (1,1) currents in almost complex manifolds deepened his interest in analytic problems arising from calibrated geometry and its connections to gauge theory. Despite important progress in this area in recent years it remains a vast source of open problems; such problems are a central component of this semester's program.

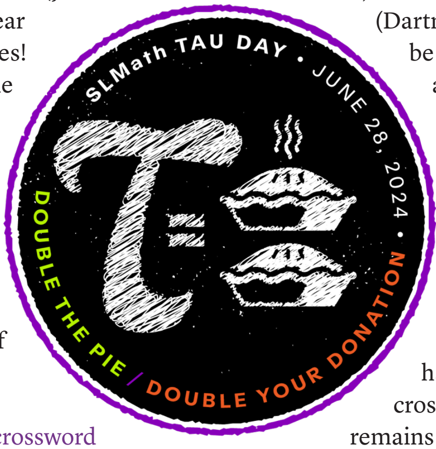
Tristan has found analysis problems arising in geometry to be particularly fascinating, because without a sufficient apprehension of the underlying geometry it is not even possible to understand what the key analytic difficulties that one must resolve are. His current work developing the analytic aspects of min-max theory for Legendrian surfaces being the latest instance of such a geometric problem that forces one to confront new analytic challenges.

While Tristan has previously visited SLMath (MSRI as it was in 2003) for a week-long workshop, this is his first semester-long visit. He is keenly looking forward to entering into the spirit of open collaboration that SLMath encourages and thereby being exposed to new ideas, future research directions and new research collaborators.

— Mark Haskins and Costante Bellettini

Celebrating Tau Day 2024

Our summer fundraising campaign celebrating Tau Day, 6.28 (June 28th), returned this year with double the puzzles! Thank you to everyone who made SLMath's Tau Day 2024 a success: In 24 hours, we raised over $54,000 towards our Annual Fund, which supports all aspects of our mission.



The annual Tau Day crossword was compiled by former SLMath postdoc **Melissa Zhang** (University of California, Davis); the winners are announced at right.

There was also a special problem, "The Blob," contributed by **Peter Winkler** (Dartmouth College), who will be at SLMath in Spring 2025 as a member of the *Probability and Statistics of Discrete Structures* program). The Blob is restated at the bottom of the Puzzles Column on page 19.

You can still try your hand at this year's crossword puzzle; the Blob remains unsolved, so proofs are still being accepted. Revisit Tau Day at tinyurl.com/tau-day-2024.

Tau Day Crossword Winners!

Grand Prize Winner
Yuval Wigderson (ETH Zürich)

2nd and 3rd Places
Katherine Paur (The Nueva School)
Daniel Ruberman (Brandeis University)

We'd also like to extend special thanks to MSRI–UP 2024 scholars Javier Garcia, Kimberly Lopez, and Russell Martinez, who observed correctly that in the blob problem, the critical fence-building speed is at most tau. (In fact it is at most two, but its exact value is still unknown.)

IBM Research, SLMath, and Duality Group Host Hudson Forum

Over the past several decades, the digital revolution, underpinned by advances in the mathematical, physical, and computer sciences, has generated profound societal and economic transformations. Yet much remains to be accomplished for humankind to achieve its full potential. In 2022, Duality Group, SLMath, and IBM partnered to host the inaugural Hudson Forum to convene experts and thinkers to share their vision on the frontiers of science and technology that will have a major impact on society in the coming decades.

Returning in September 2024, co-hosts **Darío Gil** (IBM Senior Vice President and Director of Research), **Dario Villani** (CEO and Co-Founder of Duality Group and Qognitive), and SLMath Director and University of Washington Professor of Mathematics **Tatiana Toro** welcomed leaders from academia, industry, and government to IBM Research's New York headquarters, diving into a range of deep scientific topics.

PHOTOS COURTESY OF IBM RESEARCH

Mathematician **Ingrid Daubechies** (Duke University) gave the keynote address discussing her own career and the continuing importance of mathematics in science, technology and culture. The panels and discussions that followed focused on some of the biggest current topics in computing and science, from the state of generative AI and gene editing, to neurotechnology and the future of quantum computers. Videos of all this year's talks and panels can be viewed at www.youtube.com/@Hudsonforum. For details of the presenters, visit hudsonforum.org/speakers.

Photos from left: Meta's Yann LeCun explains his theory for the future of AI models; Fyodor Urnov shows stats of how many fewer incidents patients with sickle cell disease have had since going through a CRISPR-based treatment.

The Puzzles *Column*

Joe Buhler and Tanya Khovanova

1 Is there a function from the real line $\mathbb{R}$ to real 3-space $\mathbb{R}^3$ that is injective (does not cross itself) and continuous, and has a closed image?

2 Divide a 30–60–90 triangle (that is, half of an equilateral triangle) into two 30–60–90 triangles (of different sizes) by dropping a perpendicular from the right-angled corner to the opposite side. Put the resulting two pieces together to form a shape that has a line of symmetry. (FYI: There are two solutions.)

Comment: This problem is from Daniel Bell.

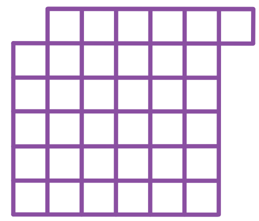
3 A random integer n is chosen from 0 through 100, inclusive (all 101 such integers are equally likely). A bag is filled with n blue marbles and $(100-n)$ red marbles, and then thoroughly mixed. Alice draws a random marble, which turns out to be blue. What probability should she assign to the event that a second random marble will also be blue? (Alice knows that the bag has 100 marbles, and how n was chosen, but does not know n.)

Comment: Daniel Litt posted this on the social media platform X in January; for an amusing account of the ensuing discussion of the problem, see Erica Klarreich's interview with Daniel in the online magazine *Quanta*.

4 Bill and Ted play a two-game chess match in which each win is worth one point and each draw is worth half a point. If the match is tied after two games, there is a "sudden-death" playoff, which means that the first player to win a game wins the match. Bill is the stronger player, winning a game with probability 0.57 if both players are trying to win. However, in any specific game Ted can choose to play a more conservative strategy that gives him no chance of winning, but a probability 0.86 of a draw and a probability of 0.14 of losing. Show that Ted can choose his strategies to have a better than 50/50 chance of winning the match.

Comment: This appeared in *Puzzles in Math and Logic* (1970) by Aaron Friedland, and the (slightly modified) version here appeared recently in Stan Wagon's puzzle column.

5 The top row of a 6×6 grid of squares is moved one square to the right. What is the smallest n such that the resulting figure can be cut along grid lines to break it in n identical pieces so that these pieces can be reassembled into a square?

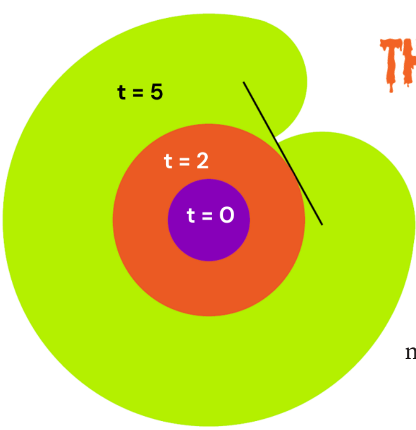


Comment: This problem appears in *A Hundred Colors of Math* (2022) by M. A. Evdokimov.

6 Finally, we are delighted to remind you of the Tau-Day "Blob" problem (below). It emerged in Peter Winkler's research with Amir Barghi, leading to further work with Jamie Schmidt which supported a conjectural critical value of the fence construction speed. However, no proof is known, and the problem is still open! Note that your solutions can be sent to the email address shown with the problem.

Segments of the fence can be built simultaneously, subject to the constraint that the total rate of growth is at most r. Peter notes that the problem can also be formulated in terms of an invisible spy who starts on the unit circle (or, if you prefer, at the origin with a 1 time unit head start); your job is to find the smallest r_0 such that the spy can definitely be corralled for any $r > r_0$.

Send your thoughts to the authors at puzzles@slmath.org. Solutions will usually be posted online before the next issue is published.



THE BLOB A Special Puzzle from Tau Day 2024

This problem was contributed by Peter Winkler for Tau Day 2024 (see page 18). Proofs are still being accepted at answers@slmath.org!

A blob, in the shape of a disk of radius 1, appears on the plane at time 0 and grows in all directions at rate 1 (so that at time 1, if unimpeded, it will be a disk of radius 2). It can be stopped only by a special kind of fence that can be manufactured at rate r (thus, at time t, the total length of all pieces of fence cannot exceed rt). What is the least real number r_0 such that if $r > r_0$, the blob can eventually be fenced in and the world saved?

Opening doors to mathematics at SLMath. Institute staff join Fall 2024 postdocs and visitors.

Call for ***Proposals***

All proposals can be submitted to the Directorate or any member of the Scientific Advisory Committee with a copy to proposals@slmath.org. For detailed information, please see the website slmath.org/proposals.

Thematic Programs

The Scientific Advisory Committee (SAC) of the Institute meets in January, May, and November each year to consider letters of intent, pre-proposals, and proposals for programs. The deadlines to submit proposals of any kind for review by the SAC are Mar 1, Oct 1, and Dec 1. Successful proposals are usually developed from the pre-proposal in a collaborative process between the proposers, the Directorate, and the SAC, and may be considered at more than one meeting of the SAC before selection. For complete details, see slmath.org/proposals-scientific-programs.

Hot Topics Workshops

Each year SLMath runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals should be received by Mar 1, Oct 1, and Dec 1 for review at the upcoming SAC meeting. See slmath.org/proposals-hot-topics-workshops.

Revisiting Fundamental Problems Workshops

The annual Revisiting Fundamental Problems Workshop addresses fundamental questions in mathematics that have, for one reason or another, been abandoned or forgotten. Proposals should be received by Mar 1, Oct 1, and Dec 1 for review at the upcoming SAC meeting. See slmath.org/proposals-revisiting-fundamental-problems-workshops.

Workshop on Critical Issues in Math Education

The Critical Issues in Mathematics Education (CIME) workshop series addresses critical issues in mathematics education today. Workshops are designed to engage mathematicians from research-focused and teaching-focused institutions, mathematics education researchers, K-12 teachers, and other interested stakeholders. Proposals may be submitted throughout the year, but the standard target date is January 31. See slmath.org/proposals-cime.

Summer Graduate Schools

Every summer SLMath organizes several two-week long summer graduate workshops, both at SLMath and at other locations. Proposals must be submitted by Sep 1 of each year for review at the upcoming SAC meeting. See slmath.org/proposals-summer-graduate-schools.

Forthcoming **Workshops**

Nov 18–22, 2024: Geometry and Analysis of Special Structures on Manifolds

Dec 9–13, 2024: Hot Topics: Life after the Telescope Conjecture

Jan 23–24, 2025: Connections Workshop: Probability and Statistics of Discrete Structures

Jan 27–Jan 31, 2025: Introductory Workshop: Probability and Statistics of Discrete Structures

Feb 6–7, 2025: Connections Workshop: Extremal Combinatorics

Feb 10–14, 2025: Introductory Workshop: Graph Theory: Extremal, Probabilistic, and Structural

Mar 3–7, 2025: Hot Topics: Interactions between Harmonic Analysis, Homogeneous Dynamics, and Number Theory

Mar 17–21, 2025: Algebraic and Analytic Methods in Combinatorics

Apr 2–4, 2025: Critical Issues in Mathematics Education 2025: K–12 Mathematics Literacy for 21st-Century Citizenship

Apr 7–11, 2025: Simons Institute for the Theory of Computing and SLMath Joint Workshop: AI for Mathematics and Theoretical Computer Science

Apr 21–25, 2025: Detection, Estimation, and Reconstruction in Networks

May 7–9, 2025: 2025 Workshop on Mathematics and Racial Justice

2025 Summer Activities

Jun 15–Jul 27, 2025: MSRI–UP 2025: Quantitative Justice

Jun 10–Jul 12, 2025: Summer Research in Mathematics

Jun 24–Jul 5, 2025: ADJOINT & Self-ADJOINT

Dates TBD: May-UP (Atlanta)

2025 Summer Graduate Schools

Jun 2–27, 2025: 2025 PIMS–CRM Summer School in Probability (Vancouver, Canada)

Jun 2–13, 2025: Séminaire de Mathématiques Supérieures 2025: An Introduction to Recent Trends in Commutative Algebra (Toronto, Canada)

Jun 2–13, 2025: Local Limits of Random Graphs (Paris-Saclay University, France)

Jun 9–20, 2025: Statistical Optimal Transport (SLMath)

Jun 16–27, 2025: Mathematics of Climate, Sea Ice, and Polar Ecosystems (Fairbanks, Alaska)

Jun 23–Jul 3, 2025: Graphical Models in Algebraic Combinatorics (SLMath)

Jun 23–Jul 4, 2025: New Perspectives on Discriminants and Applications (Leipzig, Germany)

Jun 30–Jul 10, 2025: Noncommutative Algebraic Geometry (Antwerp, Belgium)

Jul 7–18, 2025: Computer Assisted Proofs in Applied Mathematics (SLMath)

Jul 7–18, 2025: Principled Scientific Discovery with Formal Methods (IBM, Yorktown)

Jul 14–25, 2025: Geometry and Dynamics in Higher Rank Lie Groups (UC Berkeley)

Jul 21–Aug 1, 2025: Topological and Geometric Structures in Low Dimensions (SLMath)

For more information about any of SLMath's scientific activities, please see slmath.org/scientific-activities.

Clay Senior Scholars

The Clay Mathematics Institute (www.claymath.org) has announced the 2024–25 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in topical program at an institute or university away from their home institution. Here are the Clay Senior Scholars who will work at SLMath in 2024–2025.

New Frontiers in Curvature: Flows, General Relativity, Minimal Submanifolds, and Symmetry (Fall 2024)
André Neves (University of Chicago)

Special Geometric Structures and Analysis (Fall 2024)
Vincent Guedj (Université Paul Sabatier)

Probability and Statistics of Discrete Structures (Spring 2025)
Omer Angel (University of British Columbia)

Extremal Combinatorics (Spring 2025)
Gábor Tardos (Alfréd Rényi Institute of Mathematics)

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World Premiere at JMM 2025

Journeys of Black Mathematicians: Creating Pathways

SLMath and the National Association of Mathematicians (NAM) invite all to join us at the 2025 Joint Mathematics Meetings in Seattle, WA, for the world premiere of our newest documentary film by Zala Films and director George Csicsery. The screening will be followed by a discussion of the film led by NAM President Asamoah Nkwanta (Morgan State University).

Journeys of Black Mathematicians: Creating Pathways (photo featured on the cover) is the second film in the JBM series highlighting the lives of Black pioneers in mathematics from the 20th century. It traces the impacts of segregation and prejudice, concentrating on the stories of individuals who pursued their education at academic institutions with majority White student enrollment.

The film surveys individual experiences and attitudes regarding identity, as well as educational programs aimed at increasing the number of African-Americans in STEM fields. Basic questions about the beauty and philosophical meanings of mathematics lead to discussion of careers in the field for the next generation.

All are welcome to attend (JMM registration is not required for Saturday public events). Details of the location and time are given in the box below. A public premiere in Berkeley will be held following the JMM, with details to be announced. The film is expected to premiere on US public television stations nationwide beginning in Spring 2025.

Join Us!

2025 JMM in Seattle Three events — No RSVP required

Mathematical Institutes Open House

Thursday, January 9, 2025
6:00–8:00 pm

Metropolitan Ballroom A
Sheraton Grand Seattle

SLMath (MSRI) Reception for Current and Future Donors

Friday, January 10, 2025
6:00–7:30pm

Issaquah
Sheraton Grand Seattle

Journeys of Black Mathematicians: Creating Pathways

World Premiere with the National Association of Mathematicians (NAM)

Saturday, January 11, 2025
11:30am–1:00pm

4C-3, Seattle Convention Center
Arch at 705 Pike

Questions? Contact
development@slmath.org